Combining metric and topological navigation of simulated robots

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Abstract

Mobile robotics and robot navigation is a growing area of scientific research. Robot simulators are useful designing and analizing tools of this domain.

Webots ([1]) is a well-known representant of these programs, a three-dimensional mobile robot simulator. Various guidance principles can be developed in C/C++ or Java programming language with the use of Webots controller programs.

In this paper a short overview is given about the problems arising in the process of the navigation, and a short taxonomy is presented about the possible problem solving methods ([2]). A brief introduction to the probabilistic navigation techniques concerning Kalman filter and expectation maximization is included with a special focus on occupancy grid.

Formerly I presented a metric navigation method based on occupancy grid working in the Webots simulation environment ([3]). As a continuation of that research I created an enhancement of the former processes, a hybrid metric-topological navigation mechanism. A topologic layer is introduced in the environment exploration phase replacing the older value iteration ([4]). The implementation of a topologic graph of the explorable places using the metric map enables the robot to navigate in a more efficient manner. A comparison of the pure metric and the new hybrid methods is also given.

Keywords: robot simulation, probabilistic mapping, occupancy grid, metric/topological navigation

1 Introduction

Mobile robotics can be the main propellant power of the development in the 21st century. If robots will be able to take up humans' not liked, monotone, everyday tasks then the world will change more revolutionarily than with the rise of the internet.

The frequently heard answer to the question 'When will we reach this new era?' is in ten years. The underestimation of the problem is driven by several factors, high hopes of artificial intelligence is one of them. After all, an important component of a future robot servant is a navigation module that can map and travel the environment without human aid.

In this paper I present a method for building topological navigation graph on the top of an occupancy grid in the Webots simulator. First of all I list some basic problems of navigation. Then I outline a taxonomy of probabilistic mapping methods. After that I show in brief the creation of an occupancy grid and the environment exploration with value iteration. Finally I focus on the necessary steps of the composition of the graph and the environment exploration utilizing the evolved graph.

This project is part of my Ph.D. research with the main aim of the investigation of mobile robot navigation. The primary tool for the experiments is the Webots mobile robot simulator. After I was the runner up of the 1st Artificial Life Creators Contest organized by Cyberbotics Ltd. in 1999, I won the second contest in 2000 and obtained the simulator license as the first prize. Details of the competitions are discussed in ([5]).

2 The problem of navigation

During navigation the robot tries to determine the position of important objects and itself. For this reason usually an internal map is handled. The task is twofold: the robot has to navigate among objects using the actual map while it has to refine the map with new measurements of the environment at the same time. The problem is called *simultaneous localization and mapping* in the literature ([6]). In addition to this chicken-egg task other problems also arise.

The noise accompanying movement commands and sensor measurements worsen the quality of navigation. If the noise is indepent of the position of the robot and the time of the action then more gathered information leads to convergence after a while. However in most cases noise has not got this sympathic property, for instance measurement errors accumulate.

Another problem stems from multidimensionality of the environment: a detailed two-dimensional map of a room contains many thousand elements, that increases computational costs.

Problem of data association – which is probably the most important one – raises the issue how it is possible to associate map elements of various viewpoint and various time. For instance when a robot arrives to its starting point after the exploration of a circular gallery how can it realize that the place was already visited.

Fourth navigation problem results from the changes of the environment. Static maps generated during the exploration of experimental terrains cannot hold on in real

world where people are walking, doors are opening and closing, or when the robot finds itself in a rearranged flat.

Beyond the others a well-performing navigation procedure has to work in real time and has to be trustworthy. The final goal is generality, that is to say the robot has to be capable of navigating in universal environment.

A good survey of the state of the research and technology is the DARPA Grand Challenge in the Mojave desert in March 2004. The best robot could accomplish only 5 $\%$ of the targeted task (Figure 1). This fast justifies that autonomus navigation is a really hard task.

Figure 1: An unlucky contestant

2.1 Probabilistic mapping

Navigation methods spred seriously in the nineties generally stands on probabilistic base. One of the main reasons behind this fact can be that uncertainities caused by noise are hard to handle with a monotonic map where later changes do not exist. Common property of these methods is the usage of Bayes-theorem ([7]) generating a maximum-likelihood map, that is to say they create the most probable map according to the actual data.

2.1.1 Kalman filter

The Kalman filter is an efficient resolution of a linear difference-equation well-known in other domains as well. The filter tries to find the noisy solution recursively on a discrete timescale ([8]).

First half of the problem is described below.

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}$$

Here x_k is the vector describing the state of the environment at moment k, in case of navigation, it is the position of the robot and the neighbouring objects.

x_k mainly depends on the previous state (x_{k-1}) , secondly the commanding actions (u_{k-1}), while there is a Gaussain noise as well (w_{k-1}).

The robot does not sense the state of the environment directly, rather through its sensors. It is modelled by the second equation.

$$s_k = Hx_k + v_k$$

Here s_k means the perceived environment, also biased with a Gaussian noise (v_k) .

The Kalman filter solves the above difference-equation with the knowledge of A, B, H matrices and the parameters of the noise distributions.

Positive property of the filter is the iterativity, that is to say previous calculations can be used in the actual estimation. Drawback of the method is the lack of data assocation.

2.1.2 Expectation maximization

This method is also a general tool for finding unknown parameters of a system with sampling ([9], [2]). Expectation maximization means the alternative repetition of two steps until convergence. Firstly – during navigation – the expected value of the robot position has to be calculated using the actual map and the senses.

$$Q(m|m_k) = E_{m_k}[p(x, s|m_k)|s]$$

In the second step the actual position and the perception determine the alteration of the map, namely most probably map is searched in the space of maps.

$$m_{k+1} = argmax_m Q(m|m_k)$$

Expectation maximization gives an efficent solution of data association contrary to Kalman filter, moreover it handles arbitrary noise distribution. A major disadvantage of the method is that it does not work iteratively. The map has to be prepared from scratch every time step, or an auxiliary method is necessary. Hence only offline learning is possible because of the immense calculation costs.

2.1.3 Occupancy grid

This general structure manages a tesselation of the plane or space in cells. Each cell of the occupancy grid contains a probability value which is an estimation that the represented position is occupied by some object. Advantages of this method are that it is simple to implement and the iterative work. Among drawbacks worth mentioning the precondition of independent noise and the difficulty to navigate with.

3 Previous work

My former goal was to create a metric navigation module for a modified Khepera robot in the Webots simulation environment, that is to say I focus on metric spatial properties of objects like distances, and coordinates. The developed robot has to build a cognitive map — “a view from above” — of a square-shaped room in the size of a few square meters while it visits every reachable location ([3]). Figure 2, Figure 3, Figure 4, and Figure 5 show some typical experimental area.

The adopted method of metric navigation is based on the occupancy grid model pioneered by Moravec and Elfes ([10], [11]). The important steps of the map building, in accordance with Thrun’s work ([4]), are the following:

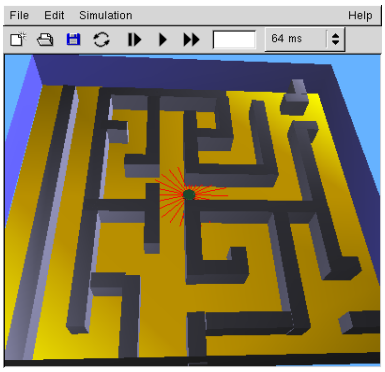

Figure 2: A maze in Webots

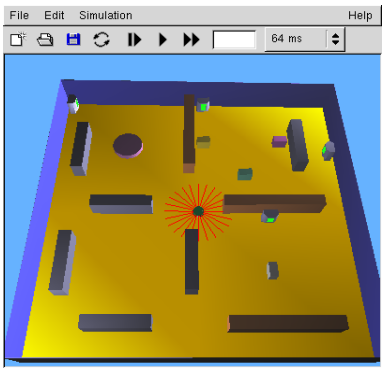

Figure 3: An office-like room

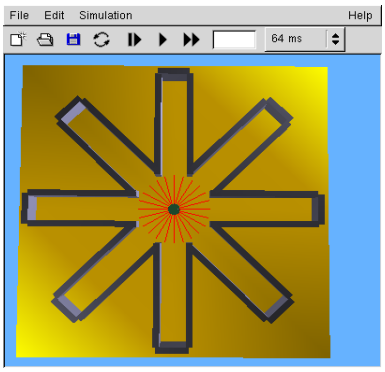

Figure 4: Radial maze

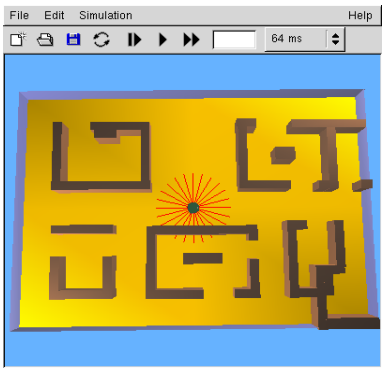

Figure 5: AAAI contest maze

- sensor interpretation
- integration over time
- pose estimation
- global grid building
- exploration with value iteration

During sensor interpretation sonar scalar values are converted to occupancy values around the robot. The integration phase summarizes different measurements. Pose estimation is omitted because the main focus of the research was creation of the occupancy map. Global grid is built in a merging process of local information.

Figure 6 shows the occupancy grid of a maze during the process of the exploration.

3.1 Value iteration

After the robot is ready to create a map of its environment, a driving force is needed to urge the robot to explore all the reachable places, otherwise it would wander randomly.

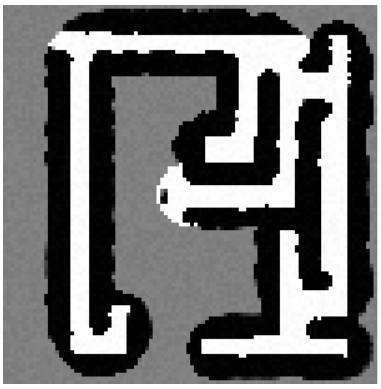

Figure 6: Occupancy grid of a maze

For this reason a variant of value iteration is implemented. This technique is well-known in the domain of reinforcement learning ([12]).

The selected algorithm helps to find the minimum cost-path to unexplored regions of the occupancy grid. A cost matrix is calculated iteratively and after convergence for every occupancy grid cell the cost of travelling to an unexplored grid cell from the actual cell is given (Figure 7, Figure 8).

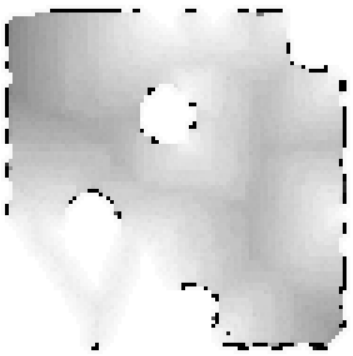

Figure 7: Cost matrix of the open area

Figure 8: Cost matrix of the maze

Exploration direction is then a resultant of the cost matrix, the actual direction of the robot, and an obstacle-avoidance behaviour.

4 Building a topological graph from occupancy grid

Exploration using value iteration is a very time-consuming task. Values of the cells of the cost matrix are calculated by a process which scans through the whole matrix many

times. Furthermore the next exploration direction is based on this gradient map and it does not necessarily take into account the constraint of the robot dynamism, sometimes resulting a fairly clumsy movement.

Accordingly it seems a natural improvement to replace the value iteration module with a topological graph. The topological graph emphasizes the links between landmarks, the possibility to move from one place to another. Graph edges represent traversable corridors of the environment and graph nodes are the crossings or end points. Navigation using the graph is much faster since its size is some order of magnitude smaller than of the cost matrix. Chapter 2 of my book ([5]) compares metric and topological navigation in detail.

There are quite many different ways of creating a navigation graph using a metric map. Skeletonization, calculating Voronoi-diagrams, matching opposite contours, sparse pixel approaches are among the possibilities ([4],[13]). In any case the occupancy grid can be viewed as a two-dimensional greyscale image of the environment, hence digital image processing methods are valid approaches ([11]).

According to [14] there are enough efficient digital image processing algorithms so for basic tasks we should not reinvent them or design new ones, this is why I use well-known procedures.

Another attitude what I consider important is that the main goal of vectorization is not to produce highest but acceptable quality vectors in the shortest amount of time.

Since I selected skeletonization, steps of the creation of topological navigation using the occupancy grid are the following:

- skeletonization
- chaining the skeleton to form edges
- graph optimization
- navigation with the graph

4.1 Skeletonization

I decided to produce the skeleton of the explored and unoccupied region of the environment. At the end of the process skeleton points are those places where the robot is hopefully not blocked by any obstacles and can reach all regions of the terrain.

For this reason I utilized medial axis transform (MAT) ([15]). An interior point of the shape belongs to the medial axis if this point lies at the same distance from two or more nearest contour points. Unfortunately one drawback of MAT appeared during my tests: medial axis of discrete objects and shapes – like the discrete occupancy grid to be projected – may be disconnected. This deficiency is not acceptable in our case since the resulting skeleton has to contain all connected routes among important places of the environment.

As a second attempt, instead of using medial axis transform, I applied a thinning algorithm to “peel the union”, in other words I iteratively shrank the object to its one pixel wide skeleton ([16]). During this process the border pixels are deleted successively while topology and morphology of the object is preserved, that is to say no pixels are deleted at the end of a line or at the connection of two regions.

The thinning algorithm works as it is described in Algorithm 1. Figure 9 shows the labeling of pixels around P_1.

P_3	P_2	P_9
P_4	P_1	P_8
P_5	P_6	P_7

Figure 9: Labeling of points in thinning

Algorithm 1 The thinning algorithm

Z0(P_1) - the number of zero to nonzero translations in the sequence $\{P_2,P_3,P_4,P_5,P_6,P_7,P_8,P_9,P_2\}$
NZ(P_1) - the number of nonzero neighbours of P_1
Steps:
1. Scan through all the points of the image.
2. Calculate Z0(P_1), NZ(P_1), Z0(P_2), Z0(P_4), for all points.
3. Delete P_1 if the following conditions simultaneously satisfied:

$$2 <= \text{NZ}(P_1) <= 6,$$
$$\text{Z0}(P_1) = 1,$$
$$P_2 * P_4 * P_8 = 0 \text{ or } \text{Z0}(P_2) \neq 1$$
$$P_2 * P_4 * P_6 = 0 \text{ or } \text{Z0}(P_4) \neq 1$$

Figure 10 and Figure 11 are examples of the result of the skeletonization process using the thinning algorithm.

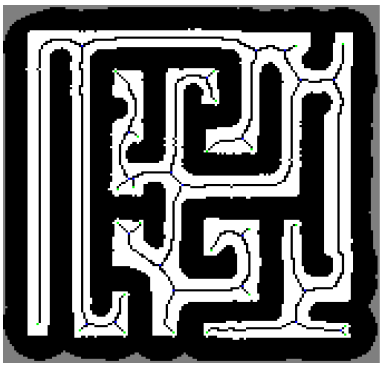

Figure 10: Skeleton of a maze

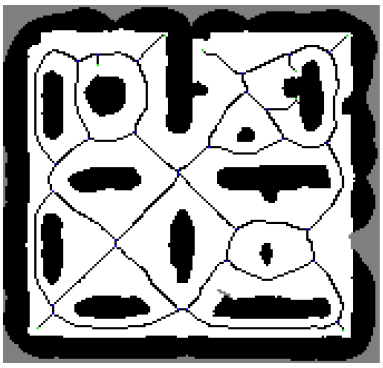

Figure 11: Skeleton of an office

4.2 Chaining

Navigation on the skeleton of the explored and unoccupied territory is possible and can be more effective than the calculation of the cost matrix of the value iteration because thinning results a data compression. Nevertheless it is advisable to use the skeleton as a basis for further processing.

Skeleton of the explored region is a set of pixels, this structure can be transformed to a graph. First of all, those points have to be determined where skeleton branches meet. These pixels are the crossing points of corridors. After I have selected the crossing points I cycle through the skeleton branches. This procedure issues in chains, what are pixel sequences from crossing point to crossing point or from crossing point to skeleton end point ([13]). Algorithm 2 reveals the main structure of the procedure.

Algorithm 2 Excerpt of the chaining algorithm

```
while there are nodes left do
  c = newChain()
  while there are non-null neighours left do
    if not found getNonNode4Neighbour(q) then
      if not found getNode4Neighbour(q) then
        if not found getNonNode8Neighbour(q) then
          if found getNode8Neighbour(q) then
            append(q,c)
          end
          endChain(c)
        else
          append(q,c)
        end
      else
        append(q,c)
        endChain(c)
      end
    else
      append(q,c)
    end
  end
end
```

The first draft of the graph is calculated during the chaining process. Skeleton crossing point and end points take part in the graph as nodes. Graph edges connect those nodes between which a chain exists.

During my investigation it turned out that the cited algorithm has two minor problems that, in special cases, corrupts the graph. On Figure 12 and Figure 13 chain creation starts from nodes (marked by 'o') and cycles through all the neighbours of the node (marked by 'x'). Non-node elements are cancelled after they take part in a chain.

First problem rises in situations similar to the one shown on Figure 12. Pixel x marked by 1 (x-1) is cancelled during the chain creation starting from x-2. In the next step – since all the neighbours of nodes have to be processed – chain creation tries to start from an already cancelled node: x-1.

Another problem is indicated on Figure 13. If the chain creation starts from x-2 then in the next step the search should turn to x-3 and chain the pixels downwards.

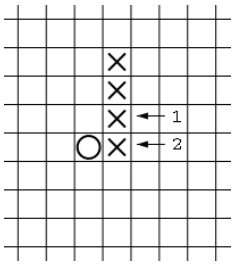

Figure 12: Chaining problem 1

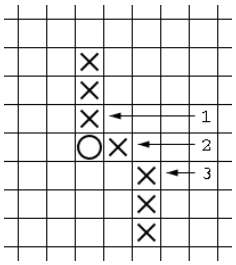

Figure 13: Chaining problem 2

However there is no explicit constraint in the algorithm to prevent the continuation after x-2 in the direction of x-1, what is obviously wrong, since it leaves x-3 without a connection to the node. After I corrected these mistakes the chaining algorithm created the draft of the navigation graph.

4.3 Graph optimization

First version of the graph is not applicable to navigate because chains may ramble far away from edges and if the robot simply follows the way of an edge it could meet with obstacles.

To cope with this problem it is possible to recursively split the edge in question and ensure that the new particles track the slues of the chain better. There are two different algorithm-family for this approximation.

Wall and Danielsson calculate the area of the surface between the edge and the chain ([17]). The iterative computation is performed by determining the sum of successive triangles. If the size of the surface exceeds a certain threshold then splitting of the edge is necessary.

Rosin and West's algorithm measures the maximal distance between the edge and the chain ([18]). This method splits the edge at its maximum deviation point recursively until all the created new edges are acceptable approximations of the chain (Figure 14).

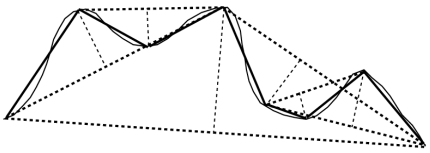

Figure 14: Splitting (taken from the slides of [13])

As a comparison of the methods [13] states that Wall and Danielsson can be implemented very efficiently but on the other hand it is less accurate than Rosin and West's method. Additionally the second mentioned algorithm may split up edges into small pieces near junctions.

Since I would like to use the topological graph for navigation at the end, it is important that edges do not cross or reach obstacles and walls. In other words fidelity of the graph to the calculated chain is important so I have chosen and implemented Rosin and West's algorithm. The procedure is described in Algorithm 3.

Algorithm 3 Algorithm of Rosin and West

```
split_edge(graph,start_point,end_point) {
  while chain is not finished do
    get_act_point(chain,act_point)
    h = height(start_point,end_point,act_point)
    if h > LIMIT then
       delete_edge_from_graph(graph,start_point,end_point)
       add_node_to_graph(graph,act_point)
       add_edge_to_graph(graph,start_point,act_point)
       add_edge_to_graph(graph,act_point,end_point)
       split_edge(start_point,act_point)
       split_edge(act_point,end_point)
    end
  end
}
```

When the recursive splitting is finished, pruning of edges is useful especially near to unexplored regions. Otherwise, if the robot simply moves to an end node where unexplored territory is nearby, then accidentaly it could run into a wall.

Figure 16 shows the optimized graph of Figure 15 after recursive edge splitting and pruning.

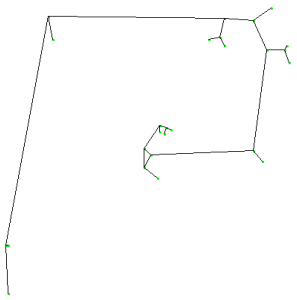

Figure 15: Graph after chaining

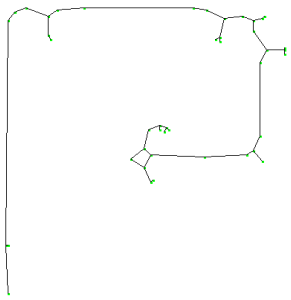

Figure 16: Optimized graph

4.4 Navigation

When creation of the graph of the explored and not occupied region is complete, the robot has to determine the next exploration direction. Generally the robot is aimed to sweep through all the reachable places of the environment. This is why those nodes of the graph where unexplored region is close can be considered as goal nodes.

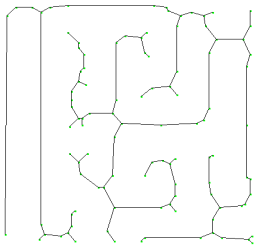

Figure 17: Navigation graph of the maze

To localize these elements I performed a general A^* algorithm ([19]). This classical algorithm finds the shortest path from the predefined start node of the graph to a goal node. Start node of the graph in our case is the actual position of the robot. The A^* algorithm then calculates the shortest path from the actual position to a node where exploration could be fruitful.

Using the shortest path as a list to be processed, the robot can turn to the next node of the graph in the list and move directly ahead while it does not reach the last node in the list.

Besides the topological graph and the A^* algorithm the final robot movement is comprised another behaviour pattern as well. The role of this normal move module is to stimulate the robot straight ahead 'en plaine air', and it also ensures obstacle avoidance motion in case of necessity. Since the generation of the topological graph is time-consuming, this job is not done continuously. When the normal move module does not explore efficiently, in other words the explored surface does not grow enough, creation of the graph takes place and navigation is governed by the A^* algorithm. This alternating comportment incorporates the advantages of the two behaviour modules.

5 Results

The navigation algorithms were tested in five different environments in several experiments from various starting points. The environments were selected to cover a wide range of possible situation that could arise during map-building.

The terrains were the following: an open area with some round obstacles, a radial maze taken from [20] well-known in cognitive map researches (Figure 4), a maze (Figure 2), an office-like room (Figure 3) which was one of the fields of the Artificial Life Creators Contest, and a labyrinth used at the 1994 AAAI autonomous mobile robot competition (Figure 5, [4]).

The open area is 1 m^2, the AAAI maze is 1.85 m^2, while the others are 2.25 m^2. Five attempts were performed in every field with both algorithm. The robot could explore all the environments by the two methods.

In the small and easily solvable open area the robot spends 8 and 6.4 minutes on an average in robot performance time using value iteration and topological graph respectively. Radial maze does not cause any difficulties for the two programs, both solves it

in around 6 minutes on an average.

The most significant advance can be reached in the office environment: the 20 minutes time drops to 12.4 minutes. In the maze the time profit is smaller: the 22 minutes of value iteration is reduced to 14.5 minutes. The AAAI contest environment is easier to solve than the maze, hence time frames of value iteration and graph navigation are 14 and 11.7 respectively.

These results are collected in Table 1.

Table 1: Time comparison of the navigation methods

	Value iteration (min)	**Topological graph (min)**
Open room	8	6.4
Radial	6.3	6
Office	20	12.4
Maze	22	14.5
AAAI contest	14	11.7

The acceleration between the two methods is a consequence of the smaller number of entities with which the algorithms have to deal (Table 2). There are between 11600 and 28900 pixels in the cost matrix of the value iteration, and the number of graph nodes are between 20 and 120, depending on the size and the complexity of the environment.

Table 2: Number of entities in the navigation methods

	Value iteration (pixels)	**Topological graph (graph nodes)**
Open room	12800	50
Radial	11600	20
Office	28900	110
Maze	28900	120
AAAI contest	23700	105

6 Conclusions

This paper presents a method to build a topological graph for navigation based on occupancy grid in the simulation environment of Webots. Besides the fact that already known algorithms are used, significantly better accomplishments related to the pure occupancy grid method justify this navigation approach.

Using topological graph instead of value iteration for the determination of exploration direction seems a beneficial modification. On one hand it approximates better the nature of the navigation. On the other hand the new algorithm performs better.

First of all, the number of manipulated entities – pixels for the value iteration, and graph nodes for the topological navigation – differ in the two approaches. This gap is more than two orders of magnitude, so the graph navigation dramatically reduces the need for resources, especially the need for memory.

Secondly, better total exploration time can be achieved with the newer control procedure. Differences in the acceleration among various test fields follow from the fact that the graph mostly helps in elongated parts of the territory and at the connections of the large spaces. Open spaces are easily explorable by random obstacle avoidance so the necessary time for open room and radial maze is not diminished essentially. For the maze, the office, and the AAAI contest environment the effects are easily recognizable, since time profit exceeds 20%.

7 Future work

There are quite many different ways of continuing the research. Some of them are mentioned below:

- Testing the algorithms in real robot.
- Higher level task can be performed by the robot after successful exploration.
- Moving around in dynamic environments is a serious challenge, this extension would make the problem more interesting.
- Using position estimation may make the robot fully automate.
- Introduction of new sensor types especially video cameras may enhance the occupancy grid creation and position estimation as well.

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